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Remarks on "Cone metric spaces and fixed point theorems of T-Kannan and T-Chatterjea contractive mappings"

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ABSTRACT

Recently, José R. Morales and Edixon Rojas [José R. Morales and Edixon Rojas, Cone metric spaces and fixed point theorems of T-Kannan contractive mappings, Int. J. Math. Anal. 4 (4) (2010) 175–184] proved fixed point theorems for T-Kannan and T-Chatterjea contractions in cone metric spaces when the underlying cone is normal. The aim of this paper is to prove this without using the normality condition. Two results for these classes of contractive mappings are also proved. Examples are given to illustrate the results.

1. Introduction and preliminaries

Ordered normed spaces, cones and topical functions have applications in applied mathematics, for instance, in using Newton's approximation method [1–5], and optimization theory [6,7]. P-metric and P-normed spaces were introduced in the mid-20th century [2]; see also [4,5] by replacing an ordered Banach space instead of the set of real numbers, as the codomain for a metric. Huang and Zhang [8] re-introduced such spaces under the name of cone metric spaces, but went further, defining convergent and Cauchy sequences in terms of interior points of the underlying cone. In such a way, nonnormal cones can be used as well (although they used only normal cones), paying attention to the fact that the Sandwich theorem and continuity of the metric may not hold. They and other authors [5,9–21,25] proved some fixed point theorems for contractive-type mappings in cone metric spaces.

Consistent with [6] (see also [2,4,8,22]), the following definitions and results will be needed in what follows.

Let E be a real Banach space. A subset P of E is called a cone whenever the following conditions hold:

(a) P is closed, nonempty and $P \neq \{\theta\}$;
(b) $a, b \in \mathbb{R}$, $a, b \geq 0$, and $x, y \in P$ imply $ax + by \in P$;
(c) $P \cap (-P) = \{\theta\}$.

Given a cone $P \subset E$, we define a partial ordering $\preceq$ with respect to P by $x \preceq y$ if and only if $y - x \in P$. We shall write $x \prec y$ to indicate that $x \preceq y$ but $x \neq y$, while $x \ll y$ will stand for $y - x \in \mathrm{int}P$ (interior of P). If $\mathrm{int}P \neq \emptyset$ then P is called a solid cone (see [4]).

There exist two kinds of cones—normal (with the normal constant K) and nonnormal ones [6].

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